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Cabibbo Angle

Chandra Raju[1]

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An expression is derived for the Cabibbo angle from a consideration of the M^2 matrices of d, s and u, c quarks. With $m_u = m_d \approx 0.3$ GeV and expressions derived earlier for m_s and m_c, the Cabibbo angle is found to be 13°42′, an excellent result.

1. INTRODUCTION

With the advent of gauge theories, there is now a great hope of unifying leptons and quarks. Recently, I (Raju, 1986a) found expressions for the masses of all the known fermions, leptons, and quarks. These expressions yield very consistent values. Apart from this dramatic success, a kind of unity in the diversity is discernible in the expressions for the masses of leptons and quarks. For example, I found that

$$m_e^2 = m_1 M_{WL} \frac{(g_V/g_A)^4_{\nu_e}}{(g_V/g_A)^4_{e\mu}} \left\{1 - \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}\right\} \tag{1}$$

$$m_\mu^2 = m_1 M_{WL} \frac{(g_V/g_A)^4_{\nu_e}}{(g_V/g_A)^4_{e\mu}} \left\{1 + \left[1 - \left(\frac{g_V}{g_A}\right)^4_{e\mu}\right]^{1/2}\right\} \tag{2}$$

where m_1 is the rest mass of the electron or muon neutrino. In other words, I arrived at equations (1) and (2) by assuming that $m_{\nu_e} = m_{\nu_\mu} = m_1$. Here M_{WL} is the mass of the charged W-boson of the standard model, and g_V and g_A are the vector and axial vector couplings of the particles indicated by the subscripts with the standard Z boson. By using the standard model prescriptions of g_V and g_A in equations (1)–(2) and with $\sin^2\theta_W = 0.2254$, we observe that equations (1) and (2) yield excellent results if $m_1 \simeq 6.7$ eV and $M_{WL} \simeq 80$ GeV. In Raju (1986a) I also proved that equations (1) and (2) yield the same results in a gauge model based on the group $SU(2)_L \times SU(2)_R \times U(1)$ provided

$$(g_V/g_A)^2_{e\mu,Z} = (g_V/g_A)^2_{e\mu,D} \tag{3}$$

[1]KSM (Osmania University), Mining Complex, Kothagudem 507 106, India.

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where the subscripts indicate the (g_V/g_A) ratio of the particles with the corresponding neutral gauge boson (Raju, 1985, 1986a,b). A relation like (3) is obviously true for neutrinos if these are strictly left-handed. Therefore, leptons as a class can be categorized as those particles for which $(g_V/g_A)^2$ with one neutral boson must be equal to that ratio with other neutral bosons of the gauge theory, whatever the gauge group. For quarks such a relation need not hold good (Raju, 1986a). In another communication (Raju, 1986c), I derived equations (1) and (2) by yet another method. I extended the results to quarks also and found that (Raju, 1986a),

$$m_c^2 = m_d M_{WL} \frac{(g_V/g_A)^4_{dsb}}{(g_V/g_A)^4_{uct}} \left\{ 1 - \left[1 - \left(\frac{g_V}{g_A} \right)^4_{uct} \right]^{1/2} \right\} \tag{4}$$

$$m_t^2 = m_d M_{WL} \frac{(g_V/g_A)^4_{dsb}}{(g_V/g_A)^4_{uct}} \left\{ 1 + \left[1 - \left(\frac{g_V}{g_A} \right)^4_{uct} \right]^{1/2} \right\} \tag{5}$$

where m_d is the mass of the down quark and g_V and g_A are the vector and axial vector couplings of the particles indicated by the subscripts with the neutral Z-boson. The striking similarity of these expressions with equations (1) and (2) should be noted.

In a similar fashion, I also found that (Raju, 1986a)

$$m_s^2 = m_u M_{WL} \frac{(g_V/g_A)^4_{uct}}{(g_V/g_A)^4_{dbs}} \left\{ 1 - \left[1 - \left(\frac{g_V}{g_A} \right)^4_{dbs} \right]^{1/2} \right\} \tag{6}$$

$$m_b^2 = m_u M_{WL} \frac{(g_V/g_A)^4_{uct}}{(g_V/g_A)^4_{dbs}} \left\{ 1 + \left[1 - \left(\frac{g_V}{g_A} \right)^4_{dbs} \right]^{1/2} \right\} \tag{7}$$

We know that

$$(g_V/g_A)^2_d = (g_V/g_A)^2_s = (g_V/g_A)^2_b = (-1 + \tfrac{4}{3}x_L)^2$$

$$(g_V/g_A)^2_u = (g_V/g_A)^2_c = (g_V/g_A)^2_t = (-1 + \tfrac{8}{3}x_L)^2$$

Here $x_L = \sin^2 \theta_W$, where θ_W is the Weinberg angle. From conventional wisdom if we assume that $m_d \simeq m_u = 0.3$ GeV and take that $M_{WL} = 80$ GeV, then we have, for $x_L = 0.2254$, $m_c = 1.7$ GeV, $m_t = 21.231$ GeV, $m_s = 0.57$ GeV, and $m_b = 2.18$ GeV. If m_u and m_d are the constituent quark masses, then the above masses are constituent quark masses. On the other hand, if m_u and m_d are current quark masses, then the above are current quark masses. In the above the various masses have the expected values except for the b-quark, which has half the expected value.

I will not go into the question of the total number of quarks. In equations (4)–(7) if we use the $(g_V/g_A)^2$ of the quarks with the neutral D-boson of $SU(2)_L \times SU(2)_R \times U(1)$ we get different numbers, and so the number of quarks must be greater.

The purpose of the present note is to derive an expression for the Cabibbo angle using equations (4)-(7). In Section 2 we obtain an expression for the Cabibbo angle and Section 3 summarizes the results.

2. THE CABIBBO ANGLE

In the most general case, the masses of the quarks are given in terms of VEVs (vacuum expectation values) because of their couplings to various Higgs bosons. Let M_1 be a mass matrix of the d, s quarks. In general M_1 will have diagonal as well as off-diagonal terms. It is dangerous to diagonalize M_1 itself. We have to diagonalize $M_1^\dagger M_1 = M_1^2$. The eigenvalues of M_1^2 must be m_d^2 and m_s^2. Suppose $R_1(\theta_1)$ is the orthogonal matrix that diagonalizes M_1^2. In a similar way, let M_2 be the mass matrix of u, c quarks. Then $M_2^2 = M_2^\dagger M_2$ will have m_u^2 and m_c^2 as its eigenvalues. Again we assume that $R_2(\theta_2)$ is an orthogonal matrix that diagonalizes M_2^2. The charged current mixing matrix which connects d-type quarks to u-type quarks is given by

$$R(\theta_C) = R_1(\theta_1) R_2^{-1}(\theta_2) \tag{8}$$

where θ_C is the Cabibbo angle.

Let

$$M_1^2 = \frac{1}{(m_d^4 + m_s^4)} \begin{pmatrix} m_d^2 m_s^2 (m_s^2 + m_d^2) & m_d^2 m_s^2 (m_s^2 - m_d^2) \\ m_d^2 m_s^2 (m_s^2 - m_d^2) & (m_d^6 + m_s^6) \end{pmatrix} \tag{9}$$

This matrix is obviously diagonalized by an orthogonal matrix, say $R_1(\theta_1)$, where

$$\tan 2\theta_1 = \frac{2 \tan \theta_1}{1 - \tan^2 \theta_1} = \frac{2(m_d^2/m_s^2)}{1 - m_d^4/m_s^4} \tag{10}$$

In other words,

$$\tan \theta_1 = m_d^2/m_s^2 \tag{11}$$

The eigenvalues of M_1^2 are m_d^2 and m_s^2.

In exactly the same way, we can show that

$$M_2^2 = \frac{1}{(m_u^4 + m_c^4)} \begin{pmatrix} m_u^2 m_c^2 (m_c^2 + m_u^2) & m_u^2 m_c^2 (m_c^2 - m_u^2) \\ m_u^2 m_c^2 (m_c^2 - m_u^2) & (m_u^6 + m_c^6) \end{pmatrix} \tag{12}$$

This is diagonalized by the orthogonal matrix $R_2(\theta_2)$, where

$$\tan \theta_2 = m_u^2/m_c^2 \tag{13}$$

The eigenvalues of the mass-squared matrix M_2^2 are m_u^2 and m_c^2. From equations (8), (11), and (13), we observe that

$$\theta_C = \theta_1 - \theta_2 \tag{14}$$

We now use the values of m_s, m_c computed earlier with $m_d \simeq m_u = 0.3$ GeV to find that

$$\theta_1 = 15°29' \tag{15}$$

and

$$\theta_2 = 1°47' \tag{16}$$

From these values, we observe that

$$\theta_C = 13°42' \qquad \text{and} \qquad \sin\theta_C = 0.2368 \tag{17}$$

which is an excellent result. The usual result that $\theta_C \simeq (m_d/m_s)^{1/2}$ is based on the diagonalization of the matrix M_1, not M_1^2, and m_u/m_c is assumed to be very small. Our value of θ_C is obtained on the basis of $m_u \simeq m_d$. These masses differ by about 4–7 MeV.

From equations (4)–(7), we observe that

$$m_d^2 = \frac{m_c^2 m_t^2}{M_{WL}^2} \frac{(g_V/g_A)_{uct}^4}{(g_V/g_A)_{dsb}^8} \tag{18}$$

$$m_u^2 = \frac{m_s^2 m_b^2}{M_{WL}^2} \frac{(g_V/g_A)_{dsb}^4}{(g_V/g_A)_{uct}^8} \tag{19}$$

From these expressions we readily note that

$$\tan\theta_1 = \frac{m_c^2 m_t^2}{m_s^2 M_{WL}^2} \frac{(g_V/g_A)_{uct}^4}{(g_V/g_A)_{dsb}^8} \tag{20}$$

$$\tan\theta_2 = \frac{m_s^2 m_b^2}{M_{WL}^2 m_c^2} \frac{(g_V/g_A)_{dsb}^4}{(g_V/g_A)_{uct}^8} \tag{21}$$

From the above relations, $\tan\theta_1$ and $\tan\theta_2$ can be found if m_s^2, m_c^2, m_t^2, and m_b^2 are known, since the $(g_V/g_A)^2$ ratio for a given species can always be found from the standard model prescription.

3. DISCUSSION

Many authors believe that $\tan\theta_C \simeq (m_d/m_s)^{1/2}$ as an established fact. The correct position is that $R(\theta_C) = R^{-1}(\theta_1)R(\theta_2)$, with these orthogonal matrices as defined in the text. We could have complicated the elements of M_1^2 or M_2^2 by utilizing equations (4) and (6) first and derived these very results [equations (4) and (6)] from the eigenvalues of M_1^2 and M_2^2. But this would complicate the matter and the physics would be lost in the wilderness.

As mentioned elsewhere (Raju, 1986a), there will be two C-quarks, two S-quarks, etc., with only a difference in masses, since $(g_V/g_A)^2$ ratios are not the same in the J_{ZL} and J_{ZR} currents. But we believe that there is only one u-quark and one d-quark, because these give rise to the proton and neutron. This shows that there must be two Cabibbo angles. The average value of these two angles can be found by using $\sin^2\theta_W = 0.25$ in the $(g_V/g_A)^2$ ratio, as this is the average of the two mixing parameters $x_L = 0.2254$ and $x_R = 0.2746$ (Raju, 1986a).

The masses of up and down quarks differ by about 4–7 MeV. The mass of the up quark is given approximately by $m_p/2.79 \simeq 336$ MeV, since the anomalous magnetic moment of these quarks is small. The mass of the strange quark is about $\frac{3}{2}m_u \simeq 510$ MeV. These masses in fact explain many facts of hadron physics (Close, 1979).

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